

APPROXIMATING THE PRECIPITABLE WATER OF THE RISING AIR PARCEL USING THE SURFACE METEOROLOGICAL ELEMENTS

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Abstract:

Atmospheric precipitable water is a good indicator of the precipitation potential and is often used in the meteorological as well as hydrological studies relating to the precipitation. Normally precipitable water (P_w) is computed by using the lower tropospheric (up to 400 hPa) temp data by plotting the temperature and dew point curves on the T/ϕ gram. The atmosphere is divided into suitable pressure levels (normally in relation to the standard atmospheric levels) and the mean mixing ratio of each small layer is then read out from the T/ϕ gram. The precipitable water of each layer is then found out by using the relationship

$$P_w \text{ (in inches)} = 0.0004 \int_{P_1}^{P_2} w \cdot dP$$

Total precipitable water is obtained by adding together the precipitable water of all the sub layers.

The method given herein allows the estimation of the precipitable water without the use of the temperature data and the T/ϕ gram. The only assumption made in the use of this method is that the surface air is assumed to rise adiabatically up to 400 hPa. The method has been computerized to compute the precipitable water for 50 mb sub layer intervals above the lifting condensation level (LCL) while the total layer below the LCL is divided into four sub layers. The method eliminates the use of T/ϕ gram and provides an extremely convenient tool for computing the P_w for any number of stations and events as needed.

Introduction:

Precipitable water is the best and most direct indicator of the atmospheric moisture, which is vital element in all the hydro-met studies.

Precipitable water in a layer of the atmosphere with pressure P_1 and P_2 at the bottom and top of the layer is given by

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$$P_w \text{ (in inches)} = 0.0004 \int_{P_1}^{P_2} w.dP \quad \dots\dots\dots(1)$$

Where w is the mixing ratio at pressure P and is given by

$$w = \frac{0.622 \times e}{P - e} \quad \dots\dots\dots(2)$$

e the vapour pressure is given by

$$e = 6.11 \times \exp\left(\frac{(17.27 \times t)}{(237.3 + t)}\right) \quad \dots\dots\dots(3)$$

t is the temperature in degrees centigrade.

Thus in the case of the upper air data being available the temperature and dew point curves are plotted and the temperature and dew point at each atmospheric pressure level is found out. Starting from surface up to 400 mb level the mean mixing ratios for each atmospheric layer of say 100 mb thickness is computed and the precipitable water for each layer is worked out as per equation (1). Total precipitable water is obtained by adding the incremental precipitable water values of all the layers.

However in case the upper air data is not available then the method of computing the precipitable water is based upon the assumption that as the air parcel rises it undergoes cooling at the dry adiabatic lapse rate until it reaches the LCL. Upon reaching the LCL further rise of the rising parcel shall be at Pseudo-adiabatic lapse rate. Thus it is assumed that the air parcel is made to rise through the atmosphere and in doing so it undergoes changes in its temperature and moisture contents in passing through the various atmospheric layers up to a level of 400 mbs. The level of 400 mb is fixed on account of the fact that due to extremely low temperature at 400mb and above the capacity of the air to hold moisture is so much reduced that it becomes negligible. Starting point in this process is the availability of three surface elements i.e., pressure, temperature and dew point. Pressure and altitude are inter related and it is often required to change one to the other during the computational process. It is thus necessary to consider first the pressure-altitude relationship.

Relationship between Pressure and Altitude:

Relationship for the pressure P at any height Z is given as

$$P = P_o \left\{ 1 - \left(\frac{BZ}{T_o} \right) \right\}^{\frac{g}{RB}} \quad \dots\dots\dots(4)$$

Where B represents temperature lapse rate and T_o and P_o are the initial temperature and pressure.

For example in case of standard atmosphere where surface pressure and temperature are taken as 1013.2 mb and 15°C respectively and a temperature lapse rate of 2° C / 1000 ft

or 0.0065/m, the height of the 850 mb layer shall be given by the above relationship expressed for the determination of Z as

$$\begin{aligned} Z &= 443018 \left[1 - \left(\frac{P}{P_o} \right)^{0.19023} \right] \\ &= 443018 \left[1 - \left(\frac{850}{1013.2} \right)^{0.19023} \right] \\ &= 1456.7m \approx 1457m \quad (\text{as given in } T / \phi \text{ gm}) \end{aligned}$$

Lapse Rate Computation for Unsaturated Layer:

As already stated the rising air column shall undergo cooling initially at dry adiabatic lapse rate of 3 °C /1000 ft or 0.0099 °C/m up to the lifting condensation level. Using the surface temperature and pressure, the saturation mixing ratio w_s is computed by the equation (2) and (3).

The prevailing mixing ratio shall be given by using the same relationship except that dew point temperature t_d is used instead of dry bulb temperature t .

Next step in the process is to compute the lifting condensation level which shall determine the top of the unsaturated layer.

Lifting Condensation Level (LCL):

As the air column rises temperature falls and the saturation mixing ratio also continues to fall until a stage comes when the saturation mixing ratio of the rising parcel become equal to that of the prevailing mixing ratio (as determined by the dew point temperature). This level, called the lifting condensation level is the top of the unsaturated layer. Using the computers, the pressure and temperature at every 10 m height interval is computed using equation (4) and the mixing ratio is determined. This shall be less than the saturation mixing ratio at the ground level. The value is compared with that of the prevailing mixing ratio as computed earlier (using the dew point temperature) for equality. This way at every 10 m height interval the computed w_s is compared with the w and the process continued until the computed w_s become equal to w . This is the level at which the lifted air parcel is fully saturated and this level is called the lifted condensation level (LCL). The layer between the surface and LCL is the unsaturated layer for which the precipitable water is required to be computed first.

Precipitable Water of the Unsaturated Layer (P_{wd}):

Precipitable water of the unsaturated layer is obtained by following the steps as given below.

Determination of the Depth of Unsaturated layer and Its Division into four Sub Layers.

Depth of the unsaturated layer is obtained by substituting the pressure at the LCL from the ground level pressure. A division of this by four shall give the four equal sub layers in terms of pressure / height.

Computing Mean Mixing Ratio for Each Sub Layer.

The mixing ratio at the bottom and top of each of the four sub layers is to be computed in order to determine the mean mixing ratio.

Since computational process moves downwards along the pseudo adiabatic curve from LCL up to the surface level, the sub layers are sequentially numbered starting from LCL level. Thus sub layer one is the layer with top at LCL and the bottom pressure given by adding 1/4th of the pressure value at the LCL to pressure at LCL. Temperature at the bottom of this layer is obtained by converting the depth of the layer in meters and then working out the total increase in the temperature (in going downwards) at the moist adiabatic lapse rate using the relationship.

$$\gamma_m = \frac{\gamma_d \left[P + \frac{0.622Le}{R_a T} \right]}{\left[P + \frac{0.622}{c_p} \left\{ e \left(c + \frac{dL}{dT} \right) + \left(\frac{R_a T}{0.622} + L + \frac{Le}{P} \right) \frac{de}{dT} \right\} \right]} \dots\dots(5)$$

Where:

- T = Temperature
- P = Pressure
- L = Latent heat of vaporization
- e_s = Saturation vapour pressure at T
- R_a = Gas constant for dry air (0.287 joules/gm/K)
- c_p = Specific heat at constant pressure for dry air (1.003 joules/gm/K)
- w = Saturation mixing ratio at T and P (gm/gm)
- c = Specific heat of liquid water (4.19 joules/gm/K)
- γ_d = Dry adiabatic lapse rate (0.0099 °C)

Vapour pressure e is obtained by the equation (3)

$\frac{de}{dt}$ and $\frac{dL}{dt}$ is obtained as follows

$$e = 6.11 \times \exp\left(\frac{17.27t}{237+t}\right)$$

Differentiating

$$\begin{aligned} \frac{de}{dt} &= \frac{(e \times 273.3 \times 17.27)}{(237.3 + t)^2} \\ &= \frac{(e \times 4098)}{(237.3 + t)^2} \dots\dots\dots(6) \end{aligned}$$

$$L = \frac{de}{dt} \times \frac{(R_v \times t^2)}{e}$$

Putting the value of $\frac{de}{dt}$ from equation (6)

$$\begin{aligned} L &= \frac{(e \times 4098)}{(237.3 + t)^2} \times \frac{(R_v \times t^2)}{e} = \frac{4098 \times R_v \times t^2}{(237.3 + t)^2} = 4098 R_v \times \frac{t^2}{(237.3 + t)^2} \\ \frac{dL}{dt} &= 4098 R_v \times \frac{\{(237.3 + t)^2 \times 2t - t^2 \times 2(237.3 + t)\}}{(237.3 + t)^4} \\ &= 4098 R_v \times \frac{\{(237.3 + t) \times 2t - 2t^2\}}{(237.3 + t)^3} \\ &= 4098 R_v \times \frac{(237.3 \times 2t)}{(237.3 + t)^3} \end{aligned}$$

R_v is a gas constant for the water vapour which is equal to 0.461 Joules/gram/K

$$\begin{aligned} \frac{dL}{dt} &= 4098 \times 0.461 \times \frac{(237.3 \times 2t)}{(237.3 + t)^3} \\ &= \frac{896604 \times t}{(237.3 + t)^3} \dots\dots\dots(7) \end{aligned}$$

Values of $\frac{de}{dt}$ and $\frac{dL}{dt}$ are obtained from equation (6) and (7) respectively and substituted in equation (5).

Solution of the equation (5) shall yield the value of γ_m (pseudo adiabatic lapse rate) for the given values of pressure and temperature.

This enables the computations of temperature at the top / bottom of the layer, whose pressure at the top/bottom is known, since we know the thickness of the layer in terms of pressure. Computations of the temperature can be done readily by knowing

the value of pseudo adiabatic lapse rate as determined by solving equation (5) for the given values of pressure at the top / bottom layer.

Thus knowing the pressure and temperature at the bottom and top of the sub layer the mixing ratios at the top and bottom are obtained using the equation (2). Mean of mixing ratios is the mean mixing ratio of the sub layer.

This way working downwards along the pseudo adiabatic lapse rate the pressure and temperatures of the subsequent layers is obtained and the mean mixing ratio for each layer is found out.

Reduction of the Mean Mixing Ratios to Cater for the Saturation Deficiency

Next step involved in this process is to reduce the mean mixing ratio of each sub layer in relation to its departure from saturation as ascertained through the mean relative humidity of each sub layer. Relative humidity at the top of the unsaturated layer is 100% while the relative humidity at the surface level is obtained by the equation.

$$(RH)_s = \frac{e}{e_s} \times 100$$

Where e and e_s are the existing and saturation vapour pressures. e and e_s are obtained using the dew point (td) and temperature (t) in the above equation.

$$e = 6.11 \times \exp\left(\frac{17.27 \times td}{237.3 + td}\right)$$

$$e_s = 6.11 \times \exp\left(\frac{17.27 \times t}{237.3 + t}\right)$$

Relative humidity at the surface RH is then subtracted from 100 to know the departure from Saturation (DS).

$$\text{Thus } DS = 100 - RH$$

Then this is divided by four to know the four equal values of the departure indicated by the symbol M, thus

$$M = \frac{DS}{4}$$

Now to obtain the mean mixing ratio of the reduction factor MRF for the layer close to the surface, which is the 4th layer; the value of the MRF₄

$$MRF_4 = \frac{(RH + M + RH)}{2 \times 100} = \frac{2RH + M}{2 \times 100}$$

Similarly the mean mixing reduction factor for the layer upwards if the layer is the layer number 3 is obtained as

$$MRF_3 = \frac{(3M + 2 \times RH)}{2 \times 100}$$

While the factor for the 2nd layer is given by

$$MRF_2 = \frac{(5M + 2 \times RH)}{2 \times 100}$$

The reduction factor for the top layer (layer number 1) is given by

$$MRF_1 = \frac{(7M + 2 \times RH)}{2 \times 100}$$

Computing the Precipitable Water for Each Sub Layer

Mean mixing ratios of the sub layers as computed under para 2 are multiplied by their respective reduction factors to obtain the mean mixing ratios corrected for the lack of saturation.

Having computed the corrected mean mixing ratios of each sub layer, the precipitable water for each sub layer is computed using the equation.

$$\text{Precipitable water } P_w \text{ (in inches)} = 0.0004 \int_{P_1}^{P_2} w \cdot dP$$

Where P_1 and P_2 are the pressure at the bottom and top of the sub layer and w is the corrected mean mixing ratio of the layer and dP is the depth of the sub layer in terms of pressure.

Precipitable water for the unsaturated layer is then obtained by adding the precipitable water for all the four sub layers.

Precipitable Water of the Saturated Layer

Thickness of the saturated layer shall be from LCL up to 400 mb. Rising parcel of the air beyond LCL shall cool at the pseudo adiabatic lapse rate as given by the equation (5).

The entire moist or saturated layer is subdivided into layers of 50 mb thickness and using the pseudo adiabatic lapse rate above LCL obtained by using equation (5), the mixing ratios for the LCL level and the level above 50 mb is found out. Mixing ratio of the 50 mb layer is the mean of the mixing ratios of the bottom and top of the 50 mb layer. This shall enable the determination of the precipitable water of the 50 mb thick layer above LCL using equation (1). This way the precipitable water for each subsequent 50 mb layer up to the level 400 mb is obtained.

Precipitable water of the moist layer shall be the sum of all the precipitable waters (P_w 's) of all the 50 mb incremental layers up to 400 mb.

Total Precipitable Water:

Total precipitable water is the sum of the precipitable waters of the dry and moist layers

$$P_w = P_{wd} + P_{ws}$$

The computational Process has been computerized and the necessary software in Fortran 77 is attached as Annexure I.

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Annexure-1

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$storage:2
$debug
c      Programme QVFn.FOR used first a= 0.0099 Laps rate
c      3Cøper Thousand Feett.
      Dimension w(600),es(600),qv(600)
      character*20 sta(600)*10,st*15,stn(600),lin
      open(1,file='normal',status='old')
      open(2,file='resn.qv',status='new')
      open(3,file='tempn',status='new')
      open(4,file='t_d_diff',status='new')
      do 50 k=1,600
      mx=mod(k,12)
      if(mx.eq.1)read(1,'(9x,a15)')st
      if(mx.eq.1)read(1,'(9x,a22)')lin
      read(1,'(1x,a9,f6.1,f8.1,f9.1)',end=51)stn(k),p1,t1,td
      tdif=t1-td
      write(4,'(5x,f9.2)')tdif
      a=.0099
      z1=10*(1013.4-p1)
      write(2,'(5x, " z1 "f9.2)')z1
      t=t1
      do 20 j=1,2
      b=17.27*t
      c=237.3+t
      d1=b/c
      es(j)=611*(2.71828)**d1
      es(j)=es(j)/100.
      qv(j)=0.622*es(j)/p1
      w(j)=0.622*es(j)/(p1-es(j))
      w(j)=w(j)*1000.
      write(2,'(5x,"p1 t es(j) w(j) ",3f15.2,f10.4)')p1,t,es(j),w(j)
      t=td
20    continue
      rti=es(2)/es(1)
      t=t1
      rd=287.0
      Ra1=Rd*(1+0.608*qv(1))
      rho1=p1/(ra1*(td+273))
      g=9.81
      z2=z1+10
      t2=t-a*(z2-z1)
      write(*,'(5x," t2 "f9.2)')t2
      b=17.27*t2
      c=237.3+t2

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d2=b/c
es(3)=611*2.71828**d2
es(3)=es(3)/100.
f=g/(a*ra1)
p2=p1*((t2+273)/(t1+273))**f
qv(3)=0.622*es(3)/p2
w(3)=0.622*es(3)/(p2-es(3))
w(3)=w(3)*1000.
Ra2=Rd+(1+0.608*qv(3))
Rho2=P2/(Ra2*(t2+273))
write(*,'(5x," ra2 "f9.2)")ra2
ram=(rho1+rho2)/2
qvm=(qv(1)+qv(3))/2
wm=(w(3)+w(1))/2
amp=ram*qvm*(z2-z1)
write(2,'(f7.0)')z1
write(2,'(5x," p1 t1 es1 w1"3f15.2,2f10.4)')p1,t,es(1),w(1)
write(2,'(f7.0)')z2
write(2,'(5x," p2 t2 es2 w2"3f15.2,2f10.4)')p2,t2,es(3),w(3)
do 10 iy=3,500
z2=10+z1
a=.0099
t3=t2-a*(z2-z1)
b=17.27*t3
c=237.3+t3
d3=b/c
es(iy)=611*2.71828**d3
es(iy)=es(iy)/100.
f=g/(a*ra1)
p3=p2*((t3+273)/(t2+273))**f
qv(iy)=0.622*es(iy)/p3
w(iy)=0.622*es(iy)/(p3-es(iy))
w(iy)=w(iy)*1000.
if(w(iy).lt.w(2))then
if(ch.eq.0)then
write(2,'(5x,"p3 t3 es3 w3 ",4f15.2)')p3,t3,es(iy),w(iy)
wm=(w(iy)+w(1))/2
write(2,'(" z2 IN METERS ",f7.0)')z2
write(2,'(5x,"lcl level reached",3f10.2)')w(iy),w(1),wm
endif
et=611*exp((17.27*t)/(237.3+t))
etd=611*exp((17.27*td)/(237.3+td))
rh=etd/et*100
if(ch.eq.0)then
rh1=rh/100.
rh=(rh+100)/2/100
rm=rh*wm

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dp=p1-p3
pw=0.0004*rm*dp
write(2,'(5x,"et etd rh ="2f9.2,f7.2,f9.2)")et,etd,rh,rm
write(2,'(5x,"in_pres new_presw pre_water "3f9.2)")p1,p3,pw
if(mx.eq.1)write(3,'(2x,a)')st
write(3,'(1x,a,4f9.2,f9.2,2f7.1)')stn(k),p1,p3,t3,pw,rh1,t1,td
endif
ch=1
endif
Ra3=Rd+(1+0.608*qv(iy))
Rho3=P3/(Ra3*(t3+273))
ram=(rho2+rho3)/2
if(iy.eq.3)i=1
qvm=(qv(i)+qv(iy))/2
i=iy
amp2=ram*qvm*(z2-z1)
tam=tam+amp2
if(iy.eq.3)then
tam=tam+amp
endif
pw=tam/25.4
t2=t3
p2=p3
rho2=rho3
z1=z2
10 continue
ch=0
50 continue
51 continue
close(2)
close(3)
stop
end

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